

MARKSCHEME

May 2012

MATHEMATICAL STUDIES

Standard Level

Paper 2

22 pages

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Paper 2 Markscheme Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

1 Abbreviations

- **M** Marks awarded for **Method**
- A Marks awarded for an **Answer** or for **Accuracy**
- **R** Marks awarded for clear **Reasoning**
- **G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, irrespective of working shown.
- **AG** Answer Given in the question and consequently, marks not awarded.
- ft Marks that can be awarded as **follow through** from previous results in the question.

2 Method of Marking

- (a) All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the A0 annotation, otherwise full annotations must be shown.
- (c) Working crossed out by the candidate should not be awarded any marks.
- (d) Where candidates have written two solutions to a question, only the first solution should be marked.
- (e) If correct working results in a correct answer but then further working is developed, full marks may **not** always be awarded. Full marks **will** be awarded if the candidate shows correct working leading to the correct answer. See also section 4(c).

Example: Calculate the gradient of the line passing through the points (5,3) and (0,9).

Markscheme		Candidates' Scripts	Marking
$ \frac{9-3}{0-5} \qquad (M1) $ Award (M1) for correct substitution in gradient	(i)	$\frac{9-3}{0-5} = -\frac{6}{5}$	(MI)
formula		Gradient is $=-\frac{6}{5}$	(A1)
6		(There is clear understand	ing of the gradient.)
$=-\frac{6}{5} \qquad (AI)$		$y = -\frac{6}{5}x + 9$	
	(ii)	$\frac{9-3}{0-5} = -\frac{6}{5}$	(MI)
		$y = -\frac{6}{5}x + 9$	<i>(A0)</i>
		(There is confusion abou	ıt what is required.)

3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '(ft)'.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (e.g. negative distances or incorrect by large order of magnitude) then the final A mark should not be awarded.
- (c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, there must be working present for that part. An isolated follow through answer, without working is regarded as incorrect and receives no marks even if it is approximately correct.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme		Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ (M1)(A1) Award (M1) for substitution in sine rule formula, (A1) for correct substitutions.	(a)	$\frac{\sin A}{4} = \frac{\sin 30}{3}$	(M1)(A0) (use of sine rule but with wrong values)
$A = 22.0^{\circ} (22.0243) (A1)(G2)$		and cannot be a	(A0) I) here was not marked (ft) warded because there was n the same question part.)
(b) $x = 7 \tan (22.0243^{\circ})$ (M1) = 2.83 (2.83163) (A1)(ft)	(b)	case (i) $x = 7 \tan 41.8^{\circ}$ = 6.26 case (ii) 6.26	(M1) (A1)(ft) (G0) since no working shown

4 Using the Markscheme

- (a) A marks are **dependent** on the preceding M mark being awarded, it is **not** possible to award (M0)(A1). Once an (M0) has been awarded, all subsequent A marks are lost in that part of the question, even if calculations are performed correctly, until the next M mark. The only exception to this will be for an answer where the accuracy is specified in the question see
 - The only exception to this will be for an answer where the accuracy is specified in the question see section 5.
- (b) A marks are **dependent** on the R mark being awarded, it is **not** possible to award (A1)(R0). Hence the (A1) cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated G in the mark scheme as an alternative to the full marks. Example (M1)(A1)(G2).

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ (M1)(A1) Award (M1) for substitution in sine	(i) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ $A = 22.0^{\circ}$	(MI)(AI) (AI)
rule formula, (A1) for correct substitutions. $A = 22.0^{\circ} (22.0243) (A1)(G2)$	(ii) $A = 22.0^{\circ}$ Note: G marks are used only if no vand the answer is correct.	(G2) vorking has been shown

(d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by 'OR' etc.

(e) Unless the question specifies otherwise, accept equivalent forms. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value (for example $\sqrt{3}$ if applicable);

the full calculator display in the form 2.83163... as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; $1 \cdot 7$; 1,7.

Different descriptions of an interval: 3 < x < 5; (3, 5); (3

Different forms of notation for set properties (e.g. complement): A'; \overline{A} ; A^c ; U - A; $(A; U \setminus A, U \setminus A, U \setminus A, U \cap A)$

Different forms of logic notation: $\neg p$; p'; \overline{p} ; \overline{p} ; \overline{p} ; $\sim p$.

 $p \Rightarrow q$; $p \rightarrow q$; $q \Leftarrow p$.

(g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through scoris to the team leader.

As from Nov 11 there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

- 1. If the candidate's unrounded answer is seen and would round to the required 3 sf answer, then award (A1) and ignore subsequent rounding.
- If the candidate's unrounded answer is not seen then award (A1) if the answer given is correctly rounded to 2 or more significant figures, otherwise (A0).
 Note: If the candidate's unrounded answer is not seen and the answer is given correct to 1 s.f (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.
- 3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarised in the table below and illustrated in the examples following.

	If candidates final answer is given							
	Exact or correct to 3 or more sf	Incorrect to 3sf	Correct to 2sf ³	Incorrect to 2sf	Correct or incorrect to 1sf			
Unrounded answer seen ¹	Award the final (A1) irrespective of correct or incorrect rounding							
Unrounded answer not seen ²	(A1)	(A0) (A1) (A0) (A0)						
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen ³						

Examples:

Markscheme			Candidates' Scripts	Marking
9.43 (9.43398)	(A1)	(i)	9.43398 is seen followed by 9; 9.4; 9.43; 9.434 etc (correctly rounded)	(AI)
		(ii)	9.43398 is seen followed by 9.433; 9.44 etc (incorrectly rounded)	(AI)
		(iii)	9.4	(A1)
		(iv)	9	(A0) (correct to 1sf)
		(v)	9.3 (incorr	(A0) ectly rounded to 2sf)
		(vi)	9.44 (incorr	(A0) ectly rounded to 3sf)

Markscheme			Candidates' Scripts	Marking
7.44 (7.43798)	(A1)	(i)	7.43798 is seen followed by 7; 7.4; 7.44; 7.438 etc (correctly rounded)	(A1)
		(ii)	7.43798 is seen followed by 7.437; 7.43 etc (incorrectly rounded)	(A1)
		(iii)	7.4	(A1)
		(iv)	7	(A0) (correct to 1sf)
		(v)	7.5 (incorr	(A0) rectly rounded to 2sf)
		(vi)	7.43 (incorn	(A0) rectly rounded to 3sf)

Example: ABC is a right angled triangle with angle $ABC = 90^{\circ}$, AC = 32 cm and AB = 30 cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Sc	ripts Marking
(a) BC = $\sqrt{32^2 - 30^2}$ (M1) Award (M1) for correct substitution in	(a) BC = $\sqrt{32^2 - 30^2}$	(M1)
Pythagoras' formula	11 (cm)	(A1)
=11.1 $(\sqrt{124},11.1355)$ (cm) (A1)	(2 sf answer only seen, but correct)
(b) Area = $\frac{1}{2} \times 30 \times 11.1355$ (MI)	(b) case (i) Area =	$\frac{1}{2} \times 30 \times 11 \qquad (MI)$
Award (M1) for correct substitution in area of triangle formula		(working shown)
	=:	(A1)(ft)
= $167(167.032)$ (cm ²) (A1)(ft)		
	case (ii) $= 1$	$(M\theta)(A\theta)(\mathbf{ft})$
	•	o working shown, the answer 11 is as a ft, so no marks awarded here)

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

e.g. chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi squared	(a) 7.7	(G2)
7.68 (7.67543) (A2)	(b) 7.67	(G1)
	(c) 7.6	(G1)
	(d) 8	(G0)
	(e) 7	(G0)
	(e) 7.66	$(G\theta)$

Regression line

Markscheme		Candidates' Scripts	Marking
y = 0.888x + 13.5 (A2) ($y = 0.887686x + 13.4895$)	(a)	y = 0.89x + 13	(G2) (both accepted)
If an answer is not in the form of an equation award at most $(A1)(A0)$.	(b)	y = 0.88x + 13	(G1) (one rounding error)
	(c)	y = 0.88x + 14	(G1) (rounding error repeated)
	(d)	(i) $y = 0.9x + 13$	
		(ii) $y = 0.8x + 13$	(G1) (1sf not accepted)
	(e)	0.88x+13 (one round	(G0) ling error and not an equation)

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
(2.06, 4.49) (A1)(A1) (2.06020, 4.49253)	(a) (2.1, 4.5)	(A1)(A1) (both accepted)
	(b) (2.0, 4.4)	(A1) (same rounding error twice)
	(c) (2.06, 4.4)	(A1) (one rounding error)
	(d) (2, 4.4) (1sf no	(A0) t accepted, one rounding error)

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (e.g. $\sqrt{13}$, $2^{\frac{2}{3}}$, $4\sqrt{5}$,) may be accepted as exact answers. All other powers (e.g. of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (ft) mark regardless of an immediately preceding (M0).

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final A mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2dp.

Markscheme	;			Candidates' Scripts	Marking
\$231.62 (231.6189)	(A1)	(i)	231.6		$(A\theta)$
		(ii)	232	(Correct roundin	(A0) g to incorrect level)
		(iii)	231.61		(A0)
		(iv)	232.00	(Parts ((A0) ii) and (iv) are both
					ing to correct level)

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final *A* mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded (A1) under the accuracy rules given in Section 5.

Example:

Markscheme				Candid	ates' Scripts	Marking
(a)	37000m^2	(A1)	(a)	$36000\mathrm{m}^2$	(Incorrect answer so u	(A0) nits not considered)
(b)	$3200 \mathrm{m}^3$	(A1)	(b)	$3200\mathrm{m}^2$		(A0) (Incorrect units)

If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

(a) 90 (A1) [1 mark]

-12-

(b) (i)
$$\frac{3}{90}(0.0\overline{3}, 0.0333, 0.0333..., 3.\overline{3}\%, 3.33\%)$$
 (A1)(ft)

Note: For the denominator follow through from their answer in part (a).

(ii)
$$\frac{53}{90}(0.5\overline{8}, 0.588..., 0.589, 58.\overline{8}\%, 58.9\%)$$
 (A1)(A1)(ft)(G2)

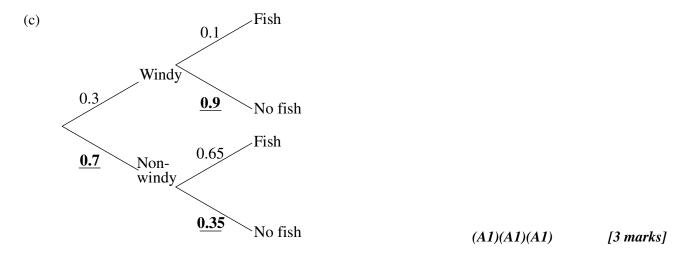
Notes: Award (A1) for the numerator. (A1)(ft) for denominator. For the denominator follow through from their answer in part (a).

(iii)
$$\frac{72}{90}$$
 (0.8, 80 %) $(AI)(ft)(AI)(ft)(G2)$

Notes: Award $(AI)(\mathbf{ft})$ for the numerator, (their part (a) -18) $(AI)(\mathbf{ft})$ for denominator. For the denominator follow through from their answer in part (a).

(iv)
$$\frac{24}{48}$$
 (0.5, 50 %) (A1)(A1)(G2) [7 marks]

Note: Award (A1) for numerator, (A1) for denominator.



Notes: Award (*A1*) for each correct entry. Tree diagram must be seen for marks to be awarded.

Question 1 continued

(d)
$$0.3 \times 0.1 = 0.03 \left(\frac{3}{100} \right)$$
 [2 marks]

Note: Award *(M1)* for **correct** product seen.

(e)
$$0.3 \times 0.1 + 0.7 \times 0.65$$
 (M1)(M1)

Notes: Award (M1) for 0.7×0.65 (or 0.455) seen, (M1) for adding their 0.03. Follow through from their answers to parts (c) and (d).

$$=0.485\left(\frac{485}{1000},\frac{97}{200}\right)$$
 (A1)(ft)(G2) [3 marks]

Note: Follow through from their tree diagram and their answer to part (d).

(f)
$$0.485 \times 0.485$$
 (M1) $0.235 \left(\frac{9409}{40000}, 0.235225 \right)$ (A1)(ft)(G2) [2 marks]

Note: Follow through from their answer to part (e).

(g)
$$\frac{0.03}{0.485}$$
 (M1)(A1)(ft)

Notes: Award (M1) for substituted conditional probability formula, (A1)(ft) for their (d) as numerator and their (e) as denominator.

$$0.0619\left(\frac{6}{97}, 0.0618556...\right)$$
 (A1)(ft)(G2) [3 marks]

Note: Follow through from their parts (d) and (e).

Total [21 marks]

QUESTION 2

(b)
$$2000+12\times600$$
 (M1)

Note: Award (M1) for addition of two correct terms.

Note: Follow through from their part (a).

(c)
$$x + 23y = 7100$$
 (A1) [1 mark]

(d)
$$x = 200, y = 300$$
 (A1)(ft)(A1)(ft)(G2) [2 marks]

(e)
$$200 + n \times 300 = 8000$$
 (M1)

Note: Award (M1) for setting up the equation. Follow through from their x and y found in part (d).

$$n = 26$$
 (A1)(ft)
26+1=27 (months) (A1)(ft)(G3)

Notes: Middle line n = 26 may be implied if correct answer given.

The final (A1)(ft) is for adding 1 to their value of n (even if it is incorrect). Follow through from their part (d).

If the final answer is not a positive integer award at most $(MI)(AI)(\mathbf{ft})(A0)$.

Award (G2) for final answer of 26.

OR

$$\frac{8000 - 7100}{300} + 24 \tag{M1)(A1)}$$

Note: Award (MI) for division of difference by their value of y, (AI) for 24 seen.

$$(A1)(ft)(G3) [3 marks]$$

Note: Follow through from their value of y.

Question 2 continued...

(f) (i)
$$2000\left(1+\frac{8}{100}\right)^3$$
 (M1)

Note: Award *(M1)* for correct substitution in compound interest formula.

2519 (euros) (A1)(G2)

Note: If the answer is not given to the nearest euro award at most (M1)(A0).

(ii)
$$2000 \left(1 + \frac{7.5}{100 \times 12}\right)^{3 \times 12}$$
 (M1)(A1)

Note: Award (MI) for substitution in compound interest formula, (AI) for correct substitutions.

2503 (euros) (A1)(G2) [5 marks]

Note: If the answer is not given to the nearest euro, award at most (M1)(A1)(A0), provided this has not been penalized in part (f)(i).

Total [14 marks]

 $(M1)(A1)(\mathrm{ft})(G2)$

QUESTION 3

(ii)

 32 ± 2

(a)	continuous		(A1)	[1 mark]
(b)	$20 < T \le 30$		(A1)	[1 mark]
(c)	15		(A1)	[1 mark]
(d)	(i) 21.5		(G2)	
	(ii) 9.21 (9	9.20597)	(G1)	[3 marks]
(e)	(i) $q = 194$		(A1)	
	(ii) $r = 200$		(A1)	[2 marks]
(g)	(i) 22.5±2	2	(A1)	

Note: Award *(M1)* for lines drawn on graph or some indication of method, follow through from their graph if working is shown.

(iii)
$$44 \pm 2$$
 (A1)(ft)

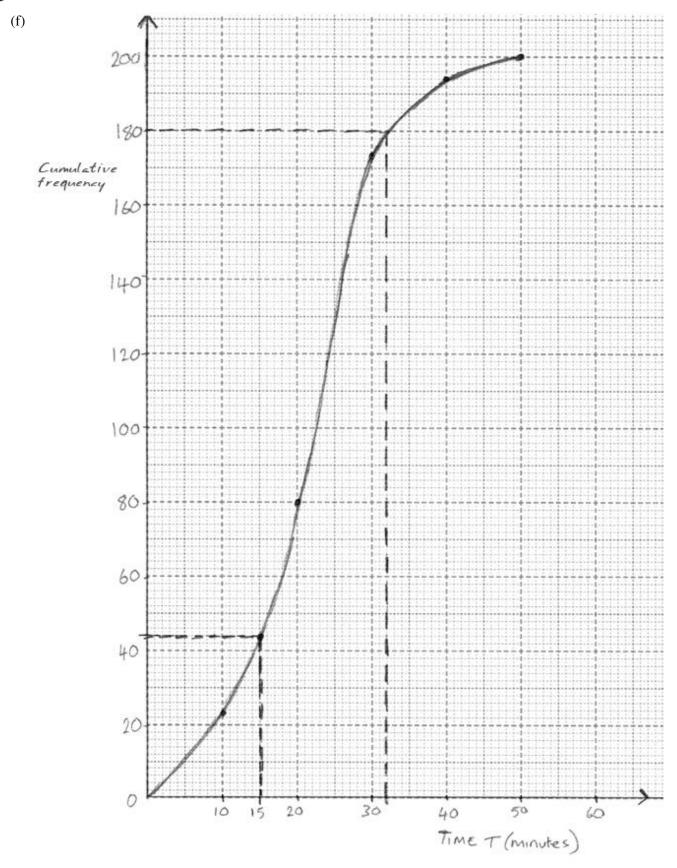
Note: Follow through from their graph if working is shown.

$$200-44=156$$
 (M1)(A1)(ft)(G2) [6 marks]

Note: Award (MI) for subtraction from 200, follow through from their graph if working is shown.

Total [18 marks]

Question 3 continued...



(A1)(A2)(ft)(A1) [4 marks]

Notes: Award (AI) for scale and axis labels, (A2)(ft) for 5 correct points, (AI)(ft) for 4 or 3 correct points, $(A\theta)$ for less than 3 correct points, (AI) for smooth curve through their points, starting at (0, 0). Follow through from their answers to parts (e)(i) and (e)(ii).

QUESTION 4 Units are required in parts (b) and (c) only.

(a) (i) $115.2 \, (m)$ (A1)

Note: Accept 115 (m)

(ii)
$$\sqrt{(146.5^2 + 115.2^2)}$$
 (M1)

Note: Award (M1) for correct substitution.

Note: Follow through from part (a)(i).

(b)
$$\frac{1}{2} \times 230.4 \times 186.368...$$
 (M1)

Note: Award (M1) for correct substitution in area of the triangle formula.

$$21500 \,\mathrm{m}^2 \quad (21469.6...\mathrm{m}^2)$$
 (A1)(ft)(G2) [2 marks]

Notes: The final answer is 21500 m²; **units are required**. Accept 21400 m² for use of 186 m and/or 115 m.

(c)
$$\frac{1}{3} \times 230.4^2 \times 146.5$$
 (M1)

Note: Award (M1) for correct substitution in volume formula.

Note: The final answer is 2590000 m³; **units are required** but do not penalise missing or incorrect units if this has already been penalised in part (b).

(d)
$$\tan^{-1} \left(\frac{146.5}{115.2} \right)$$
 (M1)

Notes: Award *(M1)* for correct substituted trig ratio. Accept alternate correct trig ratios.

$$=51.8203...=52^{\circ}$$
 (A1)(AG) [2 marks]

Notes: Both the unrounded answer and the final answer must be seen for the (AI) to be awarded.

Accept
$$51.96^{\circ} = 52^{\circ}, 51.9^{\circ} = 52^{\circ} \text{ or } 51.7^{\circ} = 52^{\circ}$$

(e)
$$128^{\circ}$$
 (A1) [1 mark]

Question 4 continued

(f)
$$\frac{186.368}{\sin 27} = \frac{x}{\sin 25}$$
 (A1)(M1)(A1)(ft)

Notes: Award (A1)(ft) for their angle MVP seen, follow through from their part (e). Award (M1) for substitution into sine formula, (A1) for correct substitutions. Follow through from their VM and their angle VMP.

$$x = 173 \text{ (m) } (173.490...)$$
 (A1)(ft)(G3) [4 marks]

Note: Accept 174 from use of 186.4.

(g)
$$VQ^2 = (186.368...)^2 + (123.490...)^2 - 2 \times (186.368...) \times (123.490...) \times \cos 128$$

(A1)(ft)(M1)(A1)(ft)

Notes: Award (AI)(ft) for 123.490...(123) seen, follow through from their x (PM) in part (f), (MI) for substitution into cosine formula, (AI)(ft) for correct substitutions. Follow through from their VM and their angle VMP.

OR

$$173.490... - 50 = 123.490... (123)$$
 (A1)(ft)
 $115.2 + 123.490... = 238.690...$ (A1)(ft)
 $VQ = \sqrt{(146.5^2 + 238.690...^2)}$ (M1)
 $VQ = 280 \text{ (m) } (280.062...)$ (A1)(ft)(G3) [4 marks]

Note: Accept 279 (m) from use of 3 significant figure answers.

Total [18 marks]

Units not required in this question.

QUESTION 5

(a) 4(2x) + 4y + 4x = 48 (M1)

Note: Award (M1) for setting up the equation.

$$12x + 4y = 48 (M1)$$

-20-

Note: Award (M1) for simplifying (can be implied).

$$y = \frac{48 - 12x}{4} \quad \mathbf{OR} \quad 3x + y = 12 \tag{A1}$$

$$y = 12 - 3x (AG) [3 marks]$$

Note: The last line must be seen for the (A1) to be awarded.

(b)
$$V = 2x \times x \times (12 - 3x)$$
 (M1)(A1)

Note: Award (MI) for substitution into volume equation, (AI) for correct substitution.

$$=24x^2-6x^3$$
 (AG) [2 marks]

Note: The last line must be seen for the (A1) to be awarded.

(c)
$$\frac{dV}{dx} = 48x - 18x^2$$
 (A1)(A1) [2 marks]

Note: Award (A1) for each correct term.

(d)
$$48x - 18x^2 = 0$$
 (M1)(M1)

Note: Award (*M1*) for using their derivative, (*M1*) for equating their answer to part (c) to 0.

OR

(M1) for sketch of
$$V = 24x^2 - 6x^3$$
, (M1) for the maximum point indicated (M1)(M1)

OR

(M1) for sketch of
$$\frac{dV}{dx} = 48x - 18x^2$$
, (M1) for the positive root indicated (M1)(M1)

$$2.67\left(\frac{24}{9}, \frac{8}{3}, 2.66666...\right)$$
 [3 marks]

Note: Follow through from their part (c).

Question 5 continued

(e)
$$V = 24 \times \left(\frac{8}{3}\right)^2 - 6 \times \left(\frac{8}{3}\right)^3$$
 (M1)

-21-

Note: Award *(M1)* for substitution of their value from part (d) into volume equation.

$$56.9 \, (\text{m}^3) \left(\frac{512}{9}, 56.8888...\right)$$
 (A1)(ft)(G2) [2 marks]

Note: Follow through from their answer to part (d).

(f) length =
$$\frac{16}{3}$$
 (A1)(ft)(G1)

Note: Follow through from their answer to part (d). Accept 5.34 from use of 2.67

height =
$$12 - 3 \times \left(\frac{8}{3}\right) = 4$$
 (M1)(A1)(ft)(G2) [3 marks]

Notes: Award (M1) for substitution of their answer to part (d), (A1)(ft) for answer.

Accept 3.99 from use of 2.67.

(g)
$$SA = 2 \times \frac{16}{3} \times 4 + 2 \times \frac{8}{3} \times 4 + 2 \times \frac{16}{3} \times \frac{8}{3}$$
 (M1)

OR

$$SA = 4\left(\frac{8}{3}\right)^2 + 6 \times \frac{8}{3} \times 4 \tag{M1}$$

Note: Award *(M1)* for substitution of their values from parts (d) and (f) into formula for surface area.

$$92.4 \,(\mathrm{m}^2) \left(92.4444...(\mathrm{m}^2)\right)$$
 (A1)

Note: Accept 92.5 (92.4622...) from use of 3 sf answers.

Question 5 continued

Number of tins =
$$\frac{92.4444...}{15 \times 4} (=1.54)$$
 (M1)

Note: Award (*MI*) for division of their surface area by 60.

2 tins required (A1)(ft) [4 marks]

Note: Follow through from their answers to parts (d) and (f).

Total [19 marks]